* Derivaties

⇒ Diff. w.r.t "x"

i) $\frac{d}{dx}$ (constant) $= 0$

ii) $\frac{d}{dx}(x) = 1$

iii) $\frac{d}{dx}(x)^n = n \cdot x^{n-1}$

iv) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

v) $\frac{d}{dx}a^x = a^x \cdot \log a$

vi) $\frac{d}{dx}e^x = e^x$

* Chain rule

⇒ Agar "x" ki jagah koi aur value aa raha hai toh waha chain rule apply karege.

eg i) $\frac{d}{dx}(x+2)^2 = 2(x+2)\cdot\frac{d}{dx}(x+2)$

ii) $\frac{d}{dx}\sqrt{x+1} = \frac{1}{2\sqrt{x+1}}\cdot\frac{d}{dx}(x+1)$

iii) $\frac{d}{dx}a^{2x} = a^{2x}\cdot\log a\cdot\frac{d}{dx}(2x)$

* Functions ⟶ Limits ⟶ Derivaties ⟶ Integration.

* Sum based on L' Hospital Rule:-

- We use Method of L'Hospital Rule whenever the question is of Indeterminate form with N^r and D^r both becoming zero after putting the value of x.

- In L'Hospital rule, we find derivate of N^r and D^r both equal number of times till D^r is $\neq 0$.

* Sums Sequence.

(A) Algebraic type

(1) , (2) , (3) , (4) , (9)
(8) , (10) , (11) , (12) , (13)
(16) , (18)

(B) Exponential type

(24) , (26) , (27) , (39) , (23)
(25)

(C) Determinant type

(5) , (6) , (7) , (17)

(D) Infinite limits

(14) , (32) , (34) , (40) , (29) ,
(28) , (30) , (33)

(E) $(\text{Variable})^{\text{variable}}$ type – Finite limits

(35) , (36)

(F) $(\text{Variable})^{\text{variable}}$ type – Infinite limits

(21)

(G) Miscellaneous

(31)

(A) Algebraic type

1) $\lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x^2 - 2x - 8}$

Method I :- Original Method

Sol^n :- Let $L = \lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x^2 - 2x - 8}$

$$= \lim_{x \to 4} \frac{x^2 - 4x + 1x - 4}{x^2 - 4x + 2x - 8}$$

$$= \lim_{x \to 4} \frac{\cancel{(x-4)}(x+1)}{\cancel{(x-4)}(x+2)}$$

$$= \lim_{x \to 4} \frac{x+1}{x+2}$$

$$= \frac{4+1}{4+2} = \boxed{\frac{5}{6}}$$ (b)

Method II :- L'Hospital Rule.

$\xrightarrow{x^n}$ $\xrightarrow{x}$ $\xrightarrow{}$ constant

Solⁿ :- Let $L = \lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x^2 - 2x - 8}$

$$= \lim_{x \to 4} \frac{2x^{2-1} - 3(1) - 0}{2x^{2-1} - 2(1) - 0} \quad \Bigg] \; \frac{d}{dx}(N^r) \;\&\; \frac{d}{dx}(D^r)$$

$$= \lim_{x \to 4} \frac{2x - 3}{2x - 2}$$

$$= \frac{2(4) - 3}{2(4) - 2} = \boxed{\frac{5}{6}} \quad (b)$$

(2) $\lim_{x \to \frac{-1}{3}} \dfrac{3x^2+7x+2}{9x^2-1}$

→ L'Hospital Rule.

Sol^n:- Let $L = \lim_{x \to -1/3} \dfrac{3x^2+7x+2}{9x^2-1}$

$x^2 \to x^n$, $x \to x$, $2 \to$ constant

$$= \lim_{x \to -1/3} \frac{3 \times 2x^{2-1} + 7(1) + 0}{9 \times 2x^{2-1} - 0}$$

$\dfrac{d}{dx}(N^r)$ & $\dfrac{d}{dx}(D^r)$

$$= \lim_{x \to -1/3} \frac{6x+7}{18x}$$

$$= \frac{6 \times \frac{-1}{3} + 7}{18 \times \frac{-1}{3}} = \frac{-2+7}{-6}$$

$$= \boxed{\frac{-5}{6}}$$ (b)

(3) $\lim_{x \to -2} \dfrac{x^3+8}{x^2+8x+12}$

Sol^n: Let $L = \lim_{x \to -2} \dfrac{x^3+8}{x^2+8x+12}$

$x^3 \to x^n$, $8 \to$ constant

x^2: x^n, $8x$: x, 12: constant

$$= \lim_{x \to -2} \frac{3x^{3-1}+0}{2x^{2-1}+8(1)+0}$$ $\frac{d}{dx}(N^r)$ & $\frac{d}{dx}(D^r)$

$$= \lim_{x \to -2} \frac{3x^2}{2x+8}$$

$$= \frac{3(-2)^2}{2(-2)+8}$$

$$= \frac{12}{4} = \boxed{3}$$ (c)

(4) $\lim_{x \to 2} \dfrac{x^3 - 2x - 4}{x^3 - x^2 - x - 2}$

Sol^n:- Let $L = \lim_{x \to 2} \dfrac{x^3 - 2x - 4}{x^3 - \underset{x^n}{x^2} - x - 2}$ $x^3 \to x^n$, $x^3 \to x^n$

$= \lim_{x \to 2} \dfrac{3x^2 - 2(1) - 0}{3x^2 - 2x - 1 - 0}$] $\frac{d}{dx}(N^r)$ & $\frac{d}{dx}(D^r)$

$= \lim_{x \to 2} \dfrac{3x^2 - 2}{3x^2 - 2x - 1}$

$= \dfrac{3(2)^2 - 2}{3(2)^2 - 2(2) - 1}$

$= \boxed{\dfrac{10}{7}}$ (d)

(9) $\lim_{x \to 3} \dfrac{\sqrt{x+2} - \sqrt{5}}{x-3}$

Sol^{n}: Let $L = \lim_{x \to 3} \dfrac{\sqrt{x+2} - \sqrt{5}}{x-3}$ → constant

constant

Chainrule

$= \lim_{x \to 3} \dfrac{\dfrac{1}{2\sqrt{x+2}} \cdot \dfrac{d}{dx}(x+2) - 0}{1 - 0}$

$\dfrac{d}{dx}(N^r)$ & $\dfrac{d}{dx}(D^r)$

+ chain rule

$= \lim_{x \to 3} \dfrac{1}{2\sqrt{x+2}} \cdot (1+0) - 0$

$= \lim_{x \to 3} \dfrac{1}{2\sqrt{x+2}}$

$= \dfrac{1}{2\sqrt{3+2}} = \boxed{\dfrac{1}{2\sqrt{5}}}$ (b)

(8) $\lim\limits_{x \to 0} \dfrac{1 - \sqrt{1-x}}{x}$

Sol^n:- Let $L = \lim\limits_{x \to 0} \dfrac{1 - \sqrt{1-x}}{x}$ chainrule

$$= \lim_{x \to 0} \frac{0 - \frac{1}{2\sqrt{1-x}} \cdot \frac{d}{dx}(1-x)}{1}$$ $\frac{d}{dx}(N^r)$ & $\frac{d}{dx}(D^r)$

$$= \lim_{x \to 0} \frac{-1}{2\sqrt{1-x}}(0-1)$$

$$= \lim_{x \to 0} \frac{-1 \times -1}{2\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{1-0}} = \boxed{\frac{1}{2}}$$ (d)

(10) $\lim_{x \to 0} \dfrac{x}{\sqrt{1+x} - \sqrt{1-x}}$

Solⁿ:- Let $L = \lim_{x \to 0} \dfrac{x}{\sqrt{1+x} - \sqrt{1-x}}$

$$= \lim_{x \to 0} \frac{1}{\frac{1}{2\sqrt{1+x}} \cdot \underbrace{\frac{d}{dx}(1+x)}_{\text{chain rule}} - \frac{1}{2\sqrt{1-x}} \cdot \underbrace{\frac{d}{dx}(1-x)}_{\text{chain rule}}}$$

$$= \lim_{x \to 0} \frac{1}{\frac{1}{2\sqrt{1+x}} \times (0+1) - \frac{1}{2\sqrt{1-x}} \times (0-1)}$$

$$= \lim_{x \to 0} \frac{1}{\frac{1}{2\sqrt{1+x}} - \frac{1 \times -1}{2\sqrt{1-x}}}$$

$$= \lim_{x \to 0} \frac{1}{\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}}}$$

$$= \frac{1}{\frac{1}{2\sqrt{1+0}} + \frac{1}{2\sqrt{1-0}}}$$

$$= \frac{1}{\frac{1}{2} + \frac{1}{2}} = \boxed{1} \quad (a)$$

(13) $\lim\limits_{x \to 2} \dfrac{\sqrt{x+7} - 3}{\sqrt{x+2} - 2}$

Sol^n:- Let $L = \lim\limits_{x \to 2} \dfrac{\sqrt{x+7} - 3}{\sqrt{x+2} - 2}$

chain rule

$$= \lim_{x \to 2} \frac{\frac{1}{2\sqrt{x+7}} \cdot \frac{d}{dx}(x+7) - 0}{\frac{1}{2\sqrt{x+2}} \cdot \frac{d}{dx}(x+2) - 0}$$

chain rule

$$= \lim_{x \to 2} \frac{\frac{1}{2\sqrt{x+7}} \times (1+0)}{\frac{1}{2\sqrt{x+2}} \times (1+0)}$$

$$= \lim_{x \to 2} \frac{\frac{1}{2\sqrt{x+7}} \begin{matrix} a \\ b \end{matrix}}{\frac{1}{2\sqrt{x+2}} \begin{matrix} c \\ d \end{matrix}} \quad \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \lim_{x \to 2} \frac{\cancel{2}\sqrt{x+2}}{\cancel{2}\sqrt{x+7}}$$

$$= \frac{\sqrt{2+2}}{\sqrt{2+7}} = \boxed{\frac{2}{3}} \qquad \text{(b)}$$

(16) $\lim_{x \to 2} \dfrac{x^2-4}{\sqrt{x+2} - \sqrt{3x-2}}$

Sol^n:- Let $L = \lim_{x \to 2} \dfrac{x^2-4}{\sqrt{x+2} - \sqrt{3x-2}}$

$$= \lim_{x \to 2} \frac{2x - 0}{\frac{1}{2\sqrt{x+2}} \cdot \frac{d}{dx}(x+2) - \frac{1}{2\sqrt{3x-2}} \cdot \frac{d}{dx}(3x-2)}$$ chain rule

$$= \lim_{x \to 2} \frac{2x}{\frac{1}{2\sqrt{x+2}} \times (1+0) - \frac{1}{2\sqrt{3x-2}} \times (3-0)}$$

$$= \lim_{x \to 2} \frac{2x}{\frac{1}{2\sqrt{x+2}} - \frac{3}{2\sqrt{3x-2}}}$$

$$= \frac{2(2)}{\frac{1}{2\sqrt{2+2}} - \frac{3}{2\sqrt{3(2)-2}}}$$

$$= \frac{4}{\frac{1}{2\times2} - \frac{3}{2\times2}}$$

$$= \frac{4}{\frac{1}{4} - \frac{3}{4}}$$

$$= \frac{4}{\frac{-2}{4}} \quad \frac{c}{d} \qquad \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \frac{4\times4}{-2} = \boxed{-8}$$ (d)

(18) $\lim_{x \to 1} \dfrac{x^{5/2} - 1}{x^{3/2} - 1}$

Sol^n:- Let $L = \lim_{x \to 1} \dfrac{x^{5/2} - 1}{x^{3/2} - 1}$ $\to x^n$

$$= \lim_{x \to 1} \frac{\frac{5}{2} \times x^{\frac{5}{2}-1} - 0}{\frac{3}{2} \times x^{3/2-1} - 0}$$

$$= \lim_{x \to 1} \frac{\frac{5}{\cancel{2}} \times x^{\frac{5-2}{2}}}{\frac{3}{\cancel{2}} \times x^{\frac{3-2}{2}}}$$

$$= \lim_{x \to 1} \frac{5x^{3/2}}{3x^{1/2}}$$

$$= \frac{5(1)^{3/2}}{3(1)^{1/2}} = \boxed{\frac{5}{3}} \quad (c)$$

(12) $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{\sqrt{x} - \sqrt{2}}$

$\underline{\underline{\text{Sol}^n}}$:- Let $L = \lim\limits_{x \to 2} \dfrac{x^3 - 8}{\sqrt{x} - \sqrt{2}}$ $\to x^n$

$= \lim\limits_{x \to 2} \dfrac{3x^2 - 0}{\frac{1}{2\sqrt{x}} - 0}$

$= \lim\limits_{x \to 2} \dfrac{3x^2}{\frac{1}{2\sqrt{x}}}$ $\dfrac{\frac{a}{b}}{\frac{c}{d}}$

$= \lim\limits_{x \to 2} 3x^2 \times 2\sqrt{x}$

$= 3(2)^2 \times 2\sqrt{2}$

$= \boxed{24\sqrt{2}}$ (c)

(11) $\lim\limits_{x \to 1} \dfrac{\sqrt{x-1}}{\sqrt{x^2-1} + \sqrt{x^3-1}}$

Solving by original Method

Sol^n:- Let $L = \lim\limits_{x \to 1} \dfrac{\sqrt{x-1}}{\sqrt{x^2-1^2} + \sqrt{x^3-1^3}}$

$\rightarrow A^2 - B^2 = (A-B)(A+B)$

$\rightarrow A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$$= \lim_{x \to 1} \frac{\sqrt{x-1}}{\sqrt{(x-1)(x+1)} + \sqrt{(x-1)(x^2+x+1)}}$$

$$= \lim_{x \to 1} \frac{\sqrt{x-1}}{\sqrt{x-1} \times \sqrt{x+1} + \sqrt{x-1}\,\sqrt{x^2+x+1}}$$

$$= \lim_{x \to 1} \frac{\sqrt{\cancel{x-1}}}{\sqrt{\cancel{x-1}}\left[\sqrt{x+1} + \sqrt{x^2+x+1}\right]}$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x+1} + \sqrt{x^2+x+1}}$$

$$= \frac{1}{\sqrt{1+1} + \sqrt{1^2+1+1}}$$

$$= \boxed{\frac{1}{\sqrt{2} + \sqrt{3}}} \quad (a)$$

(B) Exponential type :-

(24) $\lim_{x \to 0} \dfrac{3^{2x} + 2^{3x} - 2}{x}$

Solⁿ :- Let $L = \lim_{x \to 0} \dfrac{3^{2x} + 2^{3x} - 2}{x}$

$$= \lim_{x \to 0} \frac{3^{2x} \cdot \log 3 \cdot \overset{\text{chain rule}}{\frac{d}{dx}(2x)} + 2^{3x} \cdot \log 2 \cdot \overset{\text{chain rule}}{\frac{d}{dx}(3x)} - 0}{1}$$

$$= \lim_{x \to 0} 3^{2x} \cdot \log 3 \times 2(1) + 2^{3x} \cdot \log 2 \times 3(1)$$

$$= \lim_{x \to 0} 3^{2x} \cdot 2 \times \log 3 + 2^{3x} \times 3 \times \log 2$$

$$= 3^{2(0)} \times 2 \times \log 3 + 2^{3(0)} \times 3 \times \log 2$$

$$= 2\log 3 + 3\log 2$$

$$= \log 3^2 + \log 2^3 \qquad [n \cdot \log A = \log A^n]$$

$$= \log 9 + \log 8 \quad \text{(A)}$$

$$= \log (9 \times 8) \qquad [\log A + \log B = \log (A \times B)]$$

$$= \log 72 \quad \text{(B)}$$

option (c)

(26) $\lim_{x \to 0} \dfrac{a^{5x} + a^{2x} - 2}{x}$

Sol^n: Let $L = \lim_{x \to 0} \dfrac{a^{5x} + a^{2x} - 2}{x}$

chain rule chain rule

$= \lim_{x \to 0} \dfrac{a^{5x} \cdot \log a \cdot \frac{d}{dx}(5x) + a^{2x} \cdot \log a \cdot \frac{d}{dx}(2x) - 0}{1}$

$= \lim_{x \to 0} a^{5x} \cdot \log a \times 5 + a^{2x} \cdot \log a \times 2$

$= \lim_{x \to 0} a^{5x} \times 5 \times \log a + a^{2x} \times 2 \times \log a$

$= a^{5(0)} \times 5 \times \log a + a^{2(0)} \times 2 \times \log a$

$= 5 \log a + 2 \log a$

$= \log a \, (5+2)$

$= 7 \log a$ // (d)

(27) $\lim_{x\to 0} \dfrac{e^{3x} - e^{2x}}{x}$

Sol^n:- Let $L = \lim_{x\to 0} \dfrac{e^{3x} - e^{2x}}{x}$

$$= \lim_{x\to 0} \frac{e^{3x}\cdot \overset{\text{chain rule}}{\frac{d}{dx}(3x)} - e^{2x}\cdot \overset{\text{chain rule}}{\frac{d}{dx}(2x)}}{1}$$

$$= \lim_{x\to 0} e^{3x} \times (3) - e^{2x} \times (2)$$

$$= e^{3(0)} \times 3 - e^{2(0)} \times 2$$

$$= 3 - 2$$

$$= \boxed{1} \quad (b)$$

(39) $\lim_{x \to 0} \frac{9^x - 3^x}{4^x - 2^x}$

Sol^n:- Let $L = \lim_{x \to 0} \frac{9^x - 3^x}{4^x - 2^x}$

$$= \lim_{x \to 0} \frac{9^x \cdot \log 9 - 3^x \cdot \log 3}{4^x \cdot \log 4 - 2^x \cdot \log 2}$$

$$= \frac{9^{(0)} \cdot \log 9 - 3^0 \cdot \log 3}{4^{(0)} \cdot \log 4 - 2^0 \cdot \log 2}$$

$$= \frac{\log 9 - \log 3}{\log 4 - \log 2}$$

$[\log A - \log B = \log\left(\frac{A}{B}\right)]$

$$= \frac{\log\left(\frac{9}{3}\right)}{\log\left(\frac{4}{2}\right)}$$

$$= \frac{\log 3}{\log 2} \quad \text{(A)}$$

$$= \log_2 3 \quad \text{(B)}$$

$\left[\frac{\log A}{\log B} = \log_B A\right]$

option (c)

(23) $\lim_{x \to 0} \dfrac{a^{3x} - a^{2x} - a^{x} + 1}{x^2}$

Sol^n:- Let $L = \lim_{x \to 0} \dfrac{a^{3x} - a^{2x} - a^{x} + 1}{x^2}$*

$$* \quad a^{3x} - a^{2x} - a^{x} + 1 = a^{x+2x} - a^{2x} - a^{x} + 1$$

$$= \underline{a^{x} \times a^{2x} - a^{2x}} - \underline{a^{x} + 1}$$

$$= a^{2x}[a^{x} - 1] - 1[a^{x} - 1]$$

$$= [a^{x} - 1]\,[a^{2x} - 1]$$

$$= \lim_{x \to 0} \frac{[a^{x} - 1] \times [a^{2x} - 1]}{x^2}$$

$$= \lim_{x \to 0} \frac{[a^{x} - 1]}{x} \times \frac{[a^{2x} - 1]}{x}$$

$$= \lim_{x \to 0} \frac{a^{x} - 1}{x} \times \lim_{x \to 0} \frac{a^{2x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{a^{x} \cdot \log a - 0}{1} \times \lim_{x \to 0} \frac{a^{2x} \cdot \log a \cdot \frac{d}{dx}(2x) - 0}{1}$$

$$= \lim_{x \to 0} a^{x} \cdot \log a \times \lim_{x \to 0} a^{2x} \cdot \log a \times 2$$

$$= a^{0} \cdot \log a \times a^{2(0)} \cdot \log a \times 2$$

$$= \log a \times \log a \times 2$$

$$= 2\,(\log a)^2 \quad \text{(d)}$$

(25) $\lim\limits_{x \to 0} \dfrac{10^x - 2^x - 5^x + 1}{x^2}$

Sol^n:- Let $L = \lim\limits_{x \to 0} \dfrac{10^x - 2^x - 5^x + 1}{x^2}$ *

* $10^x - 2^x - 5^x + 1 = (2 \times 5)^x - 2^x - 5^x + 1$

$= 2^x \times 5^x - 2^x - 5^x + 1$

$= 2^x[5^x - 1] - 1[5^x - 1]$

$= [5^x - 1][2^x - 1]$

$= \lim\limits_{x \to 0} \dfrac{[5^x - 1][2^x - 1]}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{[5^x - 1]}{x} \times \dfrac{[2^x - 1]}{x}$

$= \lim\limits_{x \to 0} \dfrac{(5^x - 1)}{x} \times \lim\limits_{x \to 0} \dfrac{(2^x - 1)}{x}$

$= \lim\limits_{x \to 0} \dfrac{5^x \cdot \log 5 - 0}{1} \times \lim\limits_{x \to 0} \dfrac{2^x \cdot \log 2 - 0}{1}$

$= \lim\limits_{x \to 0} 5^x \cdot \log 5 \times \lim\limits_{x \to 0} 2^x \cdot \log 2$

$= 5^0 \cdot \log 5 \times 2^0 \cdot \log 2$

$= \log 5 \times \log 2$ c

(c) DETERMINANT TYPE

(5) $\displaystyle\lim_{x \to 0} \left[2 + \frac{3}{4 + \frac{5}{x}}\right]$

Sol^n:- Let $L = \displaystyle\lim_{x \to 0} \left[2 + \frac{3}{4 + \frac{5}{x}}\right]$

$$= \lim_{x \to 0} \left[2 + \frac{3}{\frac{4x + 5}{x}}\right] \qquad \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \lim_{x \to 0} \left[2 + \frac{3x}{4x + 5}\right]$$

$$= 2 + \frac{3(0)}{4(0) + 5}$$

$$= 2 + \frac{0}{5}$$

$$= 2 \quad (b)$$

(6) $$\lim_{x \to 3} \left[\frac{1}{x-3} - \frac{3}{x^2-3x} \right]$$

Solⁿ:- Let $L = \lim_{x \to 3} \left[\frac{1}{x-3} - \frac{3}{x^2-3x} \right]$

$$= \lim_{x \to 3} \left[\frac{1}{x-3} - \frac{3}{x(x-3)} \right]$$

$$= \lim_{x \to 3} \left[\frac{x}{x(x-3)} - \frac{3}{x(x-3)} \right]$$

$$= \lim_{x \to 3} \left[\frac{\cancel{(x-3)}}{x\,\cancel{(x-3)}} \right]$$

$$= \lim_{x \to 3} \left[\frac{1}{x} \right]$$

$$= \frac{1}{3}$$ (d)

(7) $\lim_{x \to 0} \left[\frac{1}{x} \left(\frac{2x+9}{x+3} - 3 \right) \right]$

Solⁿ:- Let $L = \lim_{x \to 0} \left[\frac{1}{x} \left(\frac{2x+9}{x+3} - 3 \right) \right]$

$= \lim_{x \to 0} \left[\frac{1}{x} \left(\frac{2x+9-3(x+3)}{x+3} \right) \right]$

$= \lim_{x \to 0} \left[\frac{1}{x} \left(\frac{2x+\cancel{9}-3x-\cancel{9}}{x+3} \right) \right]$

$= \lim_{x \to 0} \left[\frac{1}{\cancel{x}} \left(\frac{-\cancel{x}}{x+3} \right) \right]$

$= \lim_{x \to 0} \left[\frac{-1}{x+3} \right]$

$= \frac{-1}{0+3}$

$= \frac{-1}{3}$ (a)

(17) $\lim_{x \to 2} \left[\frac{1}{x-2} - \frac{1}{x^2-3x+2} \right]$

Sol^n:- Let $L = \lim_{x \to 2} \left[\frac{1}{x-2} - \frac{1}{x^2-3x+2} \right]$

$= \lim_{x \to 2} \left[\frac{1}{x-2} - \frac{1}{x^2-2x-1x+2} \right]$

$= \lim_{x \to 2} \left[\frac{1}{(x-2)} - \frac{1}{(x-2)(x-1)} \right]$

$= \lim_{x \to 2} \left[\frac{(x-1)}{(x-1)(x-2)} - \frac{1}{(x-2)(x-1)} \right]$

$= \lim_{x \to 2} \left[\frac{x-1-1}{(x-1)(x-2)} \right]$

$= \lim_{x \to 2} \left[\frac{\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} \right]$

$= \lim_{x \to 2} \left[\frac{1}{x-1} \right]$

$= \frac{1}{2-1}$

$= 1$ (a)

* FORMULA'S

a) $1+2+3+4+\cdots\cdots n = \boxed{\dfrac{n(n+1)}{2}}$

i.e $$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

b) $1^2+2^2+3^2+4^2+\cdots\cdots n^2 = \boxed{\dfrac{n(n+1)(2n+1)}{6}}$

i.e $$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

c) $1^3+2^3+3^3+4^3+\cdots\cdots n^3 = \boxed{\dfrac{n^2(n+1)^2}{4}}$

i.e $$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

d) $\underbrace{1+1+1+1\cdots\cdots+1}_{n\text{-times}} = \boxed{n}$

(D) SUMS BASED ON INFINITE LIMITS

***

(14) $\lim_{n\to\infty} \dfrac{1+2+3+\cdots n}{(2n+3)(n-4)}$ $\Big\}$ $\dfrac{\text{Polynomial}}{\text{Polynomial}}$ type.

Sol^n: Let $L = \lim_{n\to\infty} \dfrac{1+2+3+\cdots n}{(2n+3)(n-4)}$

$$= \lim_{n\to\infty} \dfrac{\dfrac{n(n+1)}{2}\begin{matrix}a\\b\end{matrix}}{(2n+3)(n-4)\,c} \quad \left] \dfrac{\dfrac{a}{b}}{\dfrac{c}{d}} \right.$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2(2n+3)(n-4)}$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{(4n+6)(n-4)}$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{4n^2-16n+6n-24}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{4n^2-10n-24} \quad \Big]\rightarrow \frac{\text{Polynomial}}{\text{Polynomial}}$$

Compare Degree. If N^r Degree $=$ D^r Degree then Ans = Ratio of co-efficient of highest degree $\rightarrow$ $\dfrac{1}{4}$

Dividing N^r and D^r by n^2 $\rightarrow$ highest power.

$$= \lim_{n\to\infty} \frac{\dfrac{n^2}{n^2}+\dfrac{n}{n^2}}{\dfrac{4n^2}{n^2}-\dfrac{10n}{n^2}-\dfrac{24}{n^2}}$$

$$= \lim_{n\to\infty} \frac{1 + \frac{1}{n}}{4 - 10\left(\frac{1}{n}\right) - 24\left(\frac{1}{n}\right)^2}$$

$$= \lim_{\frac{1}{n}\to 0} \frac{1 + \left(\frac{1}{n}\right)}{4 - 10\left(\frac{1}{n}\right) - 24\left(\frac{1}{n}\right)^2}$$

$$= \frac{1 + 0}{4 - 10(0) - 24(0)}$$

$$= \frac{1}{4}$$ (b)

(32) $\lim_{n\to\infty} \frac{\Sigma n^2}{n^3+1}$

Sol^n:- Let $L = \lim_{n\to\infty} \frac{\Sigma n^2}{n^3+1}$

$$= \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3+1} \quad \left.\begin{matrix} a \\ b \\ c \end{matrix}\right] \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6(n^3+1)}$$

$$= \lim_{n\to\infty} \frac{(n^2+n)(2n+1)}{6n^3+6}$$

$$= \lim_{n\to\infty} \frac{2n^3+n^2+2n^2+n}{6n^3+6}$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^2+n}{6n^3+6} \begin{matrix} \rightarrow \text{Polynomial} \\ \rightarrow \text{Polynomial} \end{matrix}$$

Compare Degree. If N^r Degree $=$ D^r Degree

then Ans = Ratio of co-efficient of highest degree.

$\downarrow$

$\frac{2}{6}$

$$= \frac{2}{6}$$

$$= \frac{1}{3} \quad (b)$$

(34) $\lim_{n \to \infty} \dfrac{\Sigma (n+1)}{2n^2+5n-6}$

<u>Sol</u>

Sol^n:- Let $L = \lim_{n \to \infty} \dfrac{\Sigma (n+1)}{2n^2+5n-6}$

$= \lim_{n \to \infty} \dfrac{\Sigma n + \Sigma 1}{2n^2+5n-6}$

$= \lim_{n \to \infty} \dfrac{\frac{n(n+1)}{2} + n}{2n^2+5n-6}$

$= \lim_{n \to \infty} \dfrac{\frac{n(n+1)+2n}{2}}{2n^2+5n-6}$ $\begin{matrix} a \\ b \\ c \end{matrix}\Bigg\} \dfrac{\frac{a}{b}}{\frac{c}{d}}$

$= \lim_{n \to \infty} \dfrac{n(n+1)+2n}{2(2n^2+5n-6)}$

$= \lim_{n \to \infty} \dfrac{n^2+n+2n}{4n^2+10n-12}$ $\begin{matrix} \rightarrow \text{Polynomial} \\ \rightarrow \text{Polynomial} \end{matrix}$

$\boxed{\text{Compare Degree. If } N^r \text{ Degree} = D^r \text{ Degree}}$

then Ans = Ratio of co-efficient of highest degree.

$\downarrow$

$\frac{1}{4}$

$= \dfrac{1}{4}$ (c)

(40) $\lim_{n\to\infty} \frac{(n+2)+(n+4)+(n+6)+\cdots+(n+2n)}{n^2}$

Sol^n: Let $L = \lim_{n\to\infty} \frac{(n+2)+(n+4)+(n+6)+\cdots+(n+2n)}{n^2}$

$$= \lim_{n\to\infty} \frac{(n+n+n+\cdots n)+(2+4+6+\cdots 2n)}{n^2}$$

$$= \lim_{n\to\infty} \frac{n(1+1+1+1\cdots+1)+2(1+2+3+\cdots n)}{n^2}$$

$$= \lim_{n\to\infty} \frac{n\,(n) + \cancel{2}\left[\frac{n(n+1)}{\cancel{2}}\right]}{n^2}$$

$$= \lim_{n\to\infty} \frac{n^2 + n^2 + n}{n^2}$$

$$= \lim_{n\to\infty} \frac{2n^2+n}{n^2} \begin{matrix}\to \text{Polynomial} \\ \to \text{Polynomial}\end{matrix}$$

Compare Degree. If N^r Degree $=$ D^r Degree

then Ans $=$ Ratio of co-efficient of highest degree.

$\frac{2}{1}$

$= 2$ (b)

(29) $$\lim_{n\to\infty} \frac{\sqrt{n^2+n-6} - \sqrt{3n^2-6}}{2n+5}$$

Sol^n: Let $L = \lim_{n\to\infty} \frac{\sqrt{n^2+n-6} - \sqrt{3n^2-6}}{2n+5}$

Dividing N^r and D^r by n → highest power

$$= \lim_{n\to\infty} \frac{\frac{\sqrt{n^2+n-6}}{n} - \frac{\sqrt{3n^2-6}}{n}}{\frac{2\cancel{n}}{\cancel{n}} + \frac{5}{n}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{n^2+n-6}{n^2}} - \sqrt{\frac{3n^2-6}{n^2}}}{2 + 5\left(\frac{1}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{\cancel{n^2}}{\cancel{n^2}} + \frac{\cancel{n}}{n^{\cancel{2}}} - \frac{6}{n^2}} - \sqrt{\frac{3\cancel{n^2}}{\cancel{n^2}} - \frac{6}{n^2}}}{2 + 5\left(\frac{1}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{1+\left(\frac{1}{n}\right)-6\left(\frac{1}{n}\right)^2} - \sqrt{3-6\left(\frac{1}{n}\right)^2}}{2+5\left(\frac{1}{n}\right)}$$

$$= \lim_{\frac{1}{n}\to 0} \frac{\sqrt{1+\left(\frac{1}{n}\right)-6\left(\frac{1}{n}\right)^2} - \sqrt{3-6\left(\frac{1}{n}\right)^2}}{2+5\left(\frac{1}{n}\right)}$$

$$= \frac{\sqrt{1+0-6(0)^2} - \sqrt{3-6(0)^2}}{2+5(0)}$$

$$= \frac{1-\sqrt{3}}{2}$$

(b)

(28) $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x+2} - \sqrt{x}\right)$

Sol^n: Let $L = \lim_{x \to \infty} \sqrt{x}\left(\sqrt{x+2} - \sqrt{x}\right)$

Rationalizing the N^r

$$= \lim_{x \to \infty} \sqrt{x}\,(\overset{(A\ -\ B)}{\sqrt{x+2} - \sqrt{x}}) \times \frac{\overset{(A\ +\ B)}{(\sqrt{x+2} + \sqrt{x})}}{(\sqrt{x+2} + \sqrt{x})}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}\left[(\sqrt{x+2})^2 - (\sqrt{x})^2\right]}{\sqrt{x+2} + \sqrt{x}} \qquad [A-B][A+B] = A^2 - B^2$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}\,[\cancel{x} + 2 - \cancel{x}]}{\sqrt{x+2} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{2\sqrt{x}}{\sqrt{x+2} + \sqrt{x}}$$

Dividing N^r and D^r by $\sqrt{x}$ → highest power

$$= \lim_{x \to \infty} \frac{\frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}}{\frac{\sqrt{x+2}}{\sqrt{x}} + \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{\frac{x+2}{x}} + 1}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{\frac{\cancel{x}}{\cancel{x}} + \frac{2}{x}} + 1}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{1 + 2\left(\frac{1}{x}\right)} + 1}$$

$$= \lim_{\frac{1}{x} \to 0} \frac{2}{\sqrt{1 + 2\left(\frac{1}{x}\right)} + 1}$$

$$= \frac{2}{\sqrt{1+2(0)} + 1}$$

$$= \frac{2}{1+1}$$

$$= \frac{2}{2}$$

$$= 1 \quad (c)$$

(30) $\lim_{x \to \infty} \left(\sqrt{x^2+5x} - x\right)$

Sol^n: Let $L = \lim_{x \to \infty} \left(\sqrt{x^2+5x} - x\right)$

Rationalizing the N^r

$$= \lim_{x \to \infty} \left(\overset{A}{\sqrt{x^2+5x}} \overset{-}{\;} \overset{B}{x}\right) \times \frac{\left(\overset{A}{\sqrt{x^2+5x}} \overset{+}{\;} \overset{B}{x}\right)}{\left(\sqrt{x^2+5x} + x\right)}$$

$$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2+5x}\right)^2 - (x)^2}{\sqrt{x^2+5x} + x} \qquad (A-B)(A+B) = A^2 - B^2$$

$$= \lim_{x \to \infty} \frac{\cancel{x^2} + 5x - \cancel{x^2}}{\sqrt{x^2+5x} + x}$$

$$= \lim_{x \to \infty} \frac{5x}{\sqrt{x^2+5x} + x}$$

Dividing N^r and D^r by x → highest power

$$= \lim_{x\to\infty} \frac{\frac{5\cancel{x}}{\cancel{x}}}{\sqrt{\frac{x^2+5x}{x}} + \frac{\cancel{x}}{\cancel{x}}}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{\frac{x^2+5x}{x^2}} + 1}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{\frac{\cancel{x^2}}{\cancel{x^2}} + \frac{5\cancel{x}}{x^{\cancel{2}}}} + 1}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{1+5\left(\frac{1}{x}\right)} + 1}$$

$$= \lim_{\frac{1}{x}\to 0} \frac{5}{\sqrt{1+5\left(\frac{1}{x}\right)} + 1}$$

$$= \frac{5}{\sqrt{1+5(0)} + 1}$$

$$= \frac{5}{1+1}$$

$$= \frac{5}{2} \quad \text{d}$$

(33) $\lim_{n\to\infty} \dfrac{\Sigma n}{\sqrt{n^2-1} \times \sqrt{n^2+3}}$
**

Sol^n:- Let $L = \lim_{n\to\infty} \dfrac{\Sigma n}{\sqrt{n^2-1} \cdot \sqrt{n^2+3}}$

$$= \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{n^2-1} \cdot \sqrt{n^2+3}} \quad \begin{matrix} a \\ b \\ c \end{matrix} \quad \left.\right] \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2(\sqrt{n^2-1}) \cdot (\sqrt{n^2+3})}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{2(\sqrt{n^2-1}) \cdot (\sqrt{n^2+3})}$$

Dividing N^r and D^r by n^2 → highest power

$$= \lim_{n\to\infty} \frac{\frac{\cancel{n^2}}{\cancel{n^2}} + \frac{n}{n^2}}{\frac{2(\sqrt{n^2-1})(\sqrt{n^2+3})}{n^2}}$$

$$= \lim_{n\to\infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \left(\frac{\sqrt{n^2-1}}{n}\right) \times \frac{\sqrt{n^2+3}}{n}}$$

$$= \lim_{n\to\infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{\frac{n^2-1}{n^2}} \times \sqrt{\frac{n^2+3}{n^2}}}$$

$$= \lim_{n\to\infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{\cancel{\frac{n^2}{n^2}} - \frac{1}{n^2}} \times \sqrt{\cancel{\frac{n^2}{n^2}} + \frac{3}{n^2}}}$$

$$= \lim_{n\to\infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{1-\left(\frac{1}{n}\right)^2} \times \sqrt{1+3\left(\frac{1}{n}\right)^2}}$$

$$= \lim_{\frac{1}{n}\to 0} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{1-\left(\frac{1}{n}\right)^2} \times \sqrt{1+3\left(\frac{1}{n}\right)^2}}$$

$$= \frac{1+(0)}{2 \times \sqrt{1-0^2} \times \sqrt{1+3(0)^2}}$$

$$= \frac{1}{2 \times 1 \times 1}$$

$$= \frac{1}{2}$$ a

Ⓔ $(\text{VARIABLE})^{\text{variable}}$ TYPE - FINITE LIMITS

* FORMULA'S

1) $\lim\limits_{x \to 0} (1+x)^{1/x} = e$

2) $\lim\limits_{x \to 0} (1+mx)^{k/x} = e^{m \times k}$

3) $\lim\limits_{x \to 0} \left(\dfrac{1+mx}{1+nx}\right)^{1/x} = e^{m-n}$

(35) $\lim_{x \to 0} \left(\frac{3-x}{3+x}\right)^{1/x}$

Sol^n:- Let $L = \lim_{x \to 0} \left(\frac{3-x}{3+x}\right)^{1/x}$

Dividing N^r and D^r by 3

$$= \lim_{x \to 0} \left(\frac{\frac{\cancel{3}}{\cancel{3}} - \frac{x}{3}}{\frac{\cancel{3}}{\cancel{3}} + \frac{x}{3}}\right)^{1/x}$$

$$= \lim_{x \to 0} \left(\frac{1 + \overbrace{\left(\frac{-1}{3}\right)}^{m} x}{1 + \underbrace{\left(\frac{1}{3}\right)}_{n} x}\right)^{1/x}$$

$\rightarrow \lim_{x \to 0} \left(\frac{1+mx}{1+nx}\right)^{1/x} = e^{m-n}$

$$= e^{-\frac{1}{3} - \frac{1}{3}}$$

$$= e^{-2/3}$$ (b)

(36) $\lim_{x \to 0} \left(\frac{3+2x}{3-x} \right)^{1/x}$

Sol^n:- Let $L = \lim_{x \to 0} \left(\frac{3+2x}{3-x} \right)^{1/x}$

Dividing N^r and D^r by 3

$$= \lim_{x \to 0} \left(\frac{\frac{\cancel{3}}{\cancel{3}} + \frac{2x}{3}}{\frac{\cancel{3}}{\cancel{3}} - \frac{x}{3}} \right)^{1/x}$$

$$= \lim_{x \to 0} \left(\frac{1 + \overbrace{\left(\frac{2}{3}\right)}^{m} x}{1 + \underbrace{\left(\frac{-1}{3}\right)}_{n} x} \right)^{1/x}$$

Now, $\lim_{x \to 0} \left(\frac{1+mx}{1+nx} \right)^{1/x} = e^{m-n}$

$$= e^{\frac{2}{3} - \left(\frac{-1}{3}\right)}$$

$$= e^{2/3 + 1/3}$$

$$= e^{3/3}$$

$$= e \quad (c)$$

(F) (VARIABLE)$^{\text{variable}}$ TYPE – INFINITE LIMITS

* FORMULA'S

1) $$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

2) $$\lim_{x \to \infty} \left(1 + \frac{m}{x}\right)^{k \cdot x} = e^{m \times k}$$

3) $$\lim_{x \to \infty} \left[\frac{1 + m\left(\frac{1}{x}\right)}{1 + n\left(\frac{1}{x}\right)}\right]^{x} = e^{m-n}$$

(21) $\lim\limits_{n\to\infty}\left(\frac{n-2}{n+3}\right)^n$

Sol^n: Let $L = \lim\limits_{n\to\infty}\left(\frac{n-2}{n+3}\right)^n$

Dividing N^r and D^r by "n"

$$= \lim_{n\to\infty}\left[\frac{\frac{\cancel{n}}{\cancel{n}} - \frac{2}{n}}{\frac{\cancel{n}}{\cancel{n}} + \frac{3}{n}}\right]^n$$

$$= \lim_{n\to\infty}\left[\frac{1+\underbrace{(-2)}_{m}\left(\frac{1}{n}\right)}{1+\underbrace{3}_{n}\left(\frac{1}{n}\right)}\right]^n$$

$$\therefore \lim_{x\to\infty}\left[\frac{1+m(1/x)}{1+n(1/x)}\right]^x = e^{m-n}$$

$$= e^{-2-3}$$

$$= e^{-5}$$

$$= \frac{1}{e^5} \quad \text{(b)}$$

(4) MISCELLANEOUS

(31) $\lim_{x \to 0} \dfrac{7x + |x|}{2x - |x|}$
*

Soln:- Let $L = \lim_{x \to 0} \dfrac{7x + |x|}{2x - |x|}$

* **Note:-** $|x| = x$, for $x > 0$

$|x| = -x$, for $x < 0$

Case I:- For $|x| = x$, when $x > 0$

$L = \lim_{x \to 0} \dfrac{7x + |x|}{2x - |x|}$

$= \lim_{x \to 0} \dfrac{7x + x}{2x - x}$

$= \lim_{x \to 0} \dfrac{8\cancel{x}}{\cancel{x}}$

$= \boxed{8}$

Case II :- For $|x| = -x$, when $x < 0$

$$L = \lim_{x \to 0} \frac{7x + |x|}{2x - |x|}$$

$$= \lim_{x \to 0} \frac{7x + (-x)}{2x - (-x)}$$

$$= \lim_{x \to 0} \frac{6\cancel{x}}{3\cancel{x}}$$

$$= \boxed{2}$$

Now $\lim_{x \to 0^+} f(x) = 8$ and $\lim_{x \to 0^-} f(x) = 2$

As $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

$\therefore$ Limit does not exist. (d)

* CONTINUITY

→ **Concept:-** A given function $f(x)$ is said to be continous if and only if its graph is continous.

But since, It is difficult to draw the graph of each and every function hence for checking the continuity of a function we follow the following methods.

* METHODS:-

(A) Denominator = 0, that value of "x" is discontinuity

(B) $\lim_{\substack{x \to a \\ x \neq a}} f(x) = f(a)_{x=a}$

limits = functions

If L.H.S = R.H.S then continous

L.H.S ≠ R.H.S then discontinous

(C) $\lim_{\substack{x \to a^- \\ x < a}} f(x) = \lim_{\substack{x \to a^+ \\ x > a}} f(x) = f(x)_{x=a}$

(A) SUMS BASED ON 1^{st} METHOD

i.e. $\boxed{\text{check Denominator} = 0}$

→ If Denominator = 0, $f(x) \rightarrow$ Discontinous.

→ If Denominator $\neq 0$, $f(x) \rightarrow$ continous.

(41) **Given:-** $f(x) = \dfrac{1}{x-2}$

To find:- Discontinous at $x = ??$

Sol^n:- $f(x) = \dfrac{1}{x-2}$

Let $x - 2 = 0$

$$\boxed{x = 2}$$

If $x = 2$ then D^r would become "0"

If $D^r = 0$ then $f(x)$ is discontinous at $x = 2$.

(C)

(42) (+) (43)

Given:- $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 3x + 2}$

Tofind:- (42) Discontinous at which value(s) of x.

(43) Continous at which value(s) of x.

Sol^n:- $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 3x + 2}$

$$= \frac{x^2 - 5x - 1x + 5}{x^2 - 2x - 1x + 2}$$

$$= \frac{(x-5)(x-1)}{(x-2)(x-1)}$$

For (42)

$f(x)$ is discontinous only when Denominator $= 0$

$\therefore (x-2)(x-1) = 0$

$x-2=0$ OR $x-1=0$

$\therefore \boxed{x=2}$ OR $\boxed{x=1}$

$\therefore$ For $x=2$ OR $x=1$, $f(x)$ will be discontinous.

(a)

For (43)

$f(x)$ is continous only when Denominator $\neq 0$

$\therefore (x-2)(x-1) \neq 0$

$x-2 \neq 0$ OR $x-1 \neq 0$

$\therefore \boxed{x \neq 2}$ OR $\boxed{x \neq 1}$

$\therefore$ $f(x)$ is continous for all values of "x" except $x=2$ OR $x=1$

(c)

(B) SUMS BASED ON 2nd METHOD

$$\Rightarrow \lim_{\substack{x \to a \\ x \neq a}} f(x) = f(x)_{x=a}$$

i.e $\boxed{\text{Limits} = \text{function}}$ then $f(x)$ is continous.

(44) Given:- i) $f(x) = \dfrac{\sqrt{x} - 2}{x - 4}$, $x \neq 4$ Limits

$f(x) = \dfrac{1}{k}$, $x = 4$ Function

ii) $f(x)$ is continous at $x = 4$

→ Limits = Function

To find:- $k = ??$

Sol^n:- As the given function is continous at $x=4$

→ Limits = Function

$\therefore \lim\limits_{x \to 4} f(x) = f(x)$

$x \neq 4 \qquad x=4$

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = f(4)$$

$$\lim_{x \to 4} \frac{\frac{1}{2\sqrt{x}} - 0}{1-0} = \frac{1}{k}$$

$$\lim_{x \to 4} \frac{1}{2\sqrt{x}} = \frac{1}{k}$$

$$\frac{1}{2\sqrt{4}} = \frac{1}{k}$$

$$\frac{1}{4} = \frac{1}{k}$$

$$\therefore \boxed{k=4} \qquad (c)$$

(46) Given:- i) $f(x) = \dfrac{x^2-16}{x-4}$, $x \neq 4$ Limits

$f(x) = 2k^2$, $x = 4$ Function

ii) $f(x)$ is continous at $x=4$

→ $\boxed{\text{Limits = Function}}$

To find:- $k = ??$

Sol^n:- As the given function is continous at $x=4$

→ $\boxed{\text{Limits = Function}}$

$$\therefore \lim_{\substack{x \to 4 \\ x \neq 4}} f(x) = \underset{x=4}{f(x)}$$

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = f(4)$$

$$\lim_{x \to 4} \frac{2x - 0}{1 - 0} = 2k^2$$

$$\lim_{x \to 4} 2x = 2k^2$$

$$\therefore 2(4) = 2k^2$$

$$\frac{8}{2} = k^2$$

$$k^2 = 4$$

$$\therefore k = \pm\sqrt{4}$$

$$\boxed{k = \pm 2} \quad a$$

(c) SUMS BASED ON 3rd METHOD

$$\boxed{\lim_{\substack{x \to a^- \\ x<a}} f(x) = \lim_{\substack{x \to a^+ \\ x>a}} f(x) = f(x)_{x=a}}$$

(47) ***

Given:- i) $f(x) = ax^2 \quad , \quad 0 \le x < 1 \longrightarrow x \to 1^-$

$f(x) = x \quad\quad 1 \le x < 2 \longrightarrow x \to 1^+ , x \to 2^-$

$f(x) = bx^3 \quad\quad 2 \le x < 3 \longrightarrow x \to 2^+$

ii) $f(x)$ is continous at $x = 1$

iii) $f(x)$ is continous at $x = 2$

$\Big] \rightarrow \lim_{x \to a^-} = \lim_{x \to a^+}$

To find:- $a = ??$, $b = ??$

$\underline{\text{Sol}^n}$:- **For $x=1$**

$\because$ $f(x)$ is continous at $x=1$

$$\therefore \lim_{\substack{x \to 1^- \\ x<1}} f(x) = \lim_{\substack{x \to 1^+ \\ x>1}} f(x)$$

$$\lim_{x \to 1} ax^2 = \lim_{x \to 1} x$$

$$a(1)^2 = 1$$

$$\therefore \boxed{a=1}$$

for $x = 2$

$\because f(x)$ is continous at $x = 2$

$$\therefore \lim_{\substack{x \to 2^- \\ x<2}} f(x) = \lim_{\substack{x \to 2^+ \\ x>2}} f(x)$$

$$\lim_{x \to 2} x = \lim_{x \to 2} bx^3$$

$$2 = b(2)^3$$

$$2 = 8b$$

$$b = \frac{2}{8}$$

$$\therefore \boxed{b = \frac{1}{4}}$$

$\therefore a = 1$ and $b = \frac{1}{4}$ (a)

(45) <u>Given</u>:- $f(x) = a - \frac{x^3}{a^2}$, $x \leq a \to \lim_{x \to a^-}$

$f(x) = a^2 - x^2$, $x > a \to \lim_{x \to a^+}$

<u>To check</u>:- $f(x)$ is continous at $x = a$

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

<u>Sol^n</u>:- i) $\lim_{\substack{x \to a^- \\ x < a}} f(x) = \lim_{x \to a^-} a - \frac{x^3}{a^2}$

$$= a - \frac{a^3}{a^2}$$

$$= a - a$$

$$= 0.$$

$$\therefore \boxed{\lim_{x \to a^-} f(x) = 0}$$

ii) $\lim_{\substack{x \to a^+ \\ x > a}} f(x) = \lim_{x \to a^+} a^2 - x^2$

$= a^2 - a^2$

$= 0$

$\therefore \boxed{\lim_{x \to a^+} f(x) = 0}$

As $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

$\therefore$ $f(x)$ is continous at $x = a$

(a)